## OVERVIEW

In this activity, students identify the paths ants can take through an ant maze, to help an aardvark catch as many ants as possible. Students begin by choosing the ant's path using a coin and evaluating how often ants come out of different holes. They then use TinkerPlots to model the ants' paths, finally coming up with a list of all possible outcomes and using this list to explain what they observed from the TinkerPlots simulation.

**Activity Time:** Two class periods

### Objectives

- Use a simulation model to collect data.
- Collect data to help draw a conclusion about which event is most likely.
- Understand that the relative frequency (and probability) of certain events is determined by the number of "paths" to get there or "ways to occur."

### Common Core Standards Addressed

Design and use a simulation to generate frequencies for compound events.

*Grade 7, Statistics and Probability Standard 8c*

Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

*Grade 7, Statistics and Probability Standard 8b*

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

*Grade 7, Statistics and Probability Standard 7b*

### Prerequisites

None

### Materials

- The Ants and the Aardvark worksheet (one per student)
- **Ants.tp**
- **Ants 2.tp** (for the extension)
- Coin (one per student pair)

## *Lesson Plan*

### DAY 1

### Introduction (5 minutes)

Hand out the worksheet, and have students read the introduction and answer Steps 1 and 2. Make sure all students understand how ants go through the maze. In particular, clear up any confusion about what it means to go up or down at an intersection.

### Group Work (10 minutes)

Once students have completed the first page of the worksheet and understand how ants go through the maze, divide them into pairs and give each pair a coin (or have them provide their own). Each pair should now send ten ants through the maze by flipping a coin and recording the results. Students should complete Steps 4–6 before coming back together as a class for a short discussion of their responses.

If you have time, have the class combine their data on the board to see the total number of ants that came out of each hole.

### Student Work at Computers (30 minutes)

Before sending students to work at computers, open the TinkerPlots document **Ants.tp** and project it for the class to see. Click the **RUN** button to run the model, showing the path of one ant at slow speed. Make sure students understand that this model is set up to do the same thing they did with the coin flips. The branches in the model send an ant up or down, so you can see the path an ant takes. The last column assigns the appropriate hole number. After the path and hole are recorded in the results table, the hole number for that ant is plotted in the graph. Once students understand how the model is set up to collect the same sort of data each pair collected with a coin, have students move to computers and begin at Step 7.

Student responses to Step 11 will give you an idea of how many students now understand that the reason holes 2 and 3 have more ants coming out is because there are more paths that lead to them.

You may want to ask students if they can tell, without tracing the path on the maze, what hole u, u, d, u, d comes out of. The number of d's in a path tells the hole number. Step 12 will help students think about this.

### DAY 2

### Discussion (15 minutes)

Steps 13–17 are designed to help students who still don't understand to begin thinking in terms of the number of paths that lead to each hole. You could have them answer these questions either as homework or at the beginning of the next class period.

After students have answered Steps 13–17, again project **Ants.tp.** Change Repeat to 200 to show 200 trials. (You'll want to increase the **Run Speed** slider!) Then click the attribute *Path* in the results table, to color the cases according to the different paths.

Help them see in the plot the number of different ways of getting to each hole number by selecting the attribute *Path*, which colors the case icons differently for each individual path. Call attention to the fact that the columns in the center have a lot more colors (paths) than the columns toward the ends, and ask students what this means. (There are many paths that go to 3, but only 1 path, for example, that goes to 5.)

### Wrap-Up (5 minutes)

After students have each written a response to Step 18, discuss their responses as a class. By now, students should recognize that holes 2 and 3 have the most ants coming out of them because there are many more paths that lead to those holes than to the other holes.

### Extension

Pose this problem to your students:

The ants going through the maze have grown lazy. They don't like to fight gravity and go up, and they would much rather go down. To make their decision, the ants spin a spinner that is divided into two sections, 25% "up" and 75% "down." Now which two holes should the aardvark stand between to catch the most ants?

The sampler in **Ants2.tp** models the decisions of the lazy ants. After students have come up with their answers (most will anticipate correctly that the ant eater should move down), either run this sampler as a class or direct students to open the file and run the simulation until they think they have an answer.

In this case, it is not just the number of ways to get there that determines the relative frequency (probability) at which the ants exit from each hole. It's also determined by the probability of choosing up or down, and those probabilities are now different. For example, u, u ,u, u, u is now much less likely than d, d, d, d, d.

To determine based on the simulation whether it is better to stand between holes 3 and 4, or between 4 and 5, requires a very large sample size. A sample size of 2500 will generally produce more cases between 3 and 4 (the correct answer) than between 4 and 5. Encourage students to keep increasing the sample size until the basic results are consistent from trial to trial—that is, until hole 3 almost always has more outcomes than hole 5, and thus it correctly appears that standing between 3 and 4 is better than standing between 4 and 5.

## ANSWERS

1. Hole 3

2. Answers will vary. Some students may think ants are equally likely to come out of each hole because they all have "an equal chance," or "because it's a matter of chance, you can't know." Other students will understand that it not likely that ants will come out of holes 0 and 5 because the ant would have to go the same direction every time. Some students may recognize it is more likely that ants come out of some of the center holes, but not be able to give a reason for this yet.

5. Answers will vary based on coin flips.

6. Students' responses will be influenced by the data. Because the sample is small, they are likely to come to different conclusions. However, students who thought all positions were equally likely will now probably say that standing between 0 and 1, or 4 and 5, is not a good idea. Ideally, some students will express the desire and need to collect more data, and you can use this as a motivation for using the sampler in TinkerPlots to collect more data quickly. (A good way to anticipate the variety of student results from 10 trials is to set the Repeat to 10 in **Ants.tp** and draw several samples, looking at the graph. The amount of variability from sample to sample is quite surprising.)

10. Student responses to this question will give you some insight into how they are thinking. Ideally, they will express some reservations in relying too heavily on the data because there is not enough of it. They may also be aware of the results of other students. Because these results will look different, students may even suggest that the data is of little value. Again, you can use these valid reservations as a motivation for collecting a lot more data.
11. With samples of 100, the position between holes 2 and 3 will almost certainly be the position with the most ants. But few students at this point are likely to explain this in terms of the number of paths, or ways, to get there. This question is intended to get them wondering why and motivate the analysis that follows.
12. The hole the ant comes out of can be determined by the number of times it chose to go down, so you can count the number of d's in a sequence to find the hole number the ant will come out of.

| **Path** | **Hole** |
|---|---|
| u, d, u, d, u | 2 |
| d, d, d, d, d | 5 |
| u, u, u, u, u | 0 |
| d, d, u, d, d | 4 |
| d, u, d, u, u | 2 |
| u, d, d, d, d | 4 |
| u, d, u, u, u | 1 |
| d, u, u, d, d | 3 |

13. d, u, u, u, u / u, d, u, u, u / u, u, d, u, u / u, u, u, d, u / u, u, u, u, d
14. d, d, d, d, u / d, d, d, u, d / d, d, u, d, d / d, u, d, d, d / u, d, d, d, d
15. d, d, u, u, u / d, u, d, u, u / d, u, u, d, u / d, u, u, u, d / u, d, d, u, u / u, d, u, d, u / u, d, u, u, d / u, u, d, d, u / u, u, d, u, d / u, u, u, d, d
16. d, d, d, u, u / d, d, u, d, u / d, d, u, u, d / d, u, d, d, u / d, u, d, u, d / d, u, u, d, d / u, d, d, d, u / u, d, d, u, d / u, d, u, d, d / u, u, d, d, d
17. The aardvark should stand between holes 2 and 3 because there are more ways for the ants to get there, so it is likely that more ants will come out of these two holes than any other pair of holes.

18. This question requires students to generalize what they learned and apply it to coin flipping and number of heads and tails. Because they began the activity by flipping coins, many students will be able to successfully answer this question. For those who are not able to generalize, you might remind them that they began by flipping coins. You could also have them imagine changing the **Ants.tp** document so that u and d were h and t, and the hole number was the number of t's.