## OVERVIEW

This activity explores the connection between multiplication and division, and between factors and multiples. Students use roughly rectangular stacks of the numbers 1 to 100 and color every multiple of a specific factor. The patterns created by the colored numbers help them identify every multiple of the factor.

The primary goal of this activity is for students to explore how they can describe whole numbers in terms of their factors. Most middle-school students have probably had experience representing multiplication with a rectangular arrangement of blocks. This activity takes advantage of that familiarity and builds on it by allowing students to quickly change the height and width of the rectangle to see what products result.

The student worksheet is relatively straightforward, giving step-by-step instructions for creating a few plots and asking specific questions about those plots. For students in grades 6–8, you may want to break away from the worksheet and encourage students to do independent explorations of the factor patterns. Students will find them compelling and can locate and describe many patterns. The extensions at the end of these activity notes offer suggestions for going even further into algebraic concepts and primes.

A secondary goal is to show how you can use TinkerPlots to explore non-statistical concepts and data. This activity in particular uses data about the characteristics of numbers to develop number sense and explore multiple representations.

**Activity Time:** One class period

### Objectives

- Recognize different representations for the same number.
- Select, apply, and translate among mathematical representations.
- Describe numbers in terms of their factors.
- Understand various meanings of multiplication and division.
- Understand the inverse relationship between multiplication and division.

### Common Core Standards Addressed

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

*Grade 4, Operations & Algebraic Thinking Standard 4*

### Prerequisites

- Students need to know basic *multiplication* and *division* of whole numbers. Preferably, students are familiar with the mathematical term *factor*.

### Materials

- Factors worksheet (one copy per student)
- **Factors.tp**

## *Lesson Plan*

### Introduction (10 minutes)

Hand out the Factors worksheet. Read through the introduction with students and check that they understand the meaning of the attributes.

### Plot and Investigate (30 minutes)

Have students move to computers and open the file **Factors.tp**.

Although the question in Step 7 may seem obvious to you, and to many students, it is a necessary building block for some students. The goal of this question is to allow students who don't see the connection between factors and multiples an opportunity to use inductive reasoning—to recognize a pattern and make a generalization. Allowing time to explore the pattern will strengthen students' reasoning skills and help them make the connection between factors and multiples. In fact, you may prefer to have students repeat Step 7 for a few other width-factor combinations before moving on to Step 8.

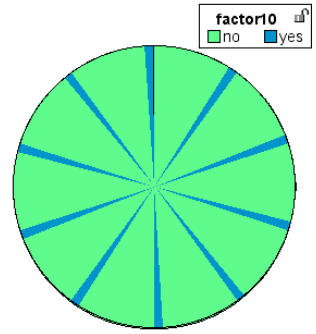
You may want to pause after Step 11 and help students see the connection to familiar rectangular diagrams for multiplication. If you look at the rectangle that is defined by 24, it is 8 wide and 3 high, or $8 \times 3 = 24$. It is because of the inverse relationship between multiplication and division that we are able to use this rectangular model to solve $24 \div 8 = 3$.

### Wrap-Up (10 minutes)

Discuss the methods of making a plot to do division problems. If you like, you can point out one way to translate between the representations in Steps 11 and 13: take the rectangle in Step 11; imagine cutting the rows into 3 separate rows; and then put the rows end to end. Show (or have students show) how to solve the division problems in Step 14 or other similar problems.

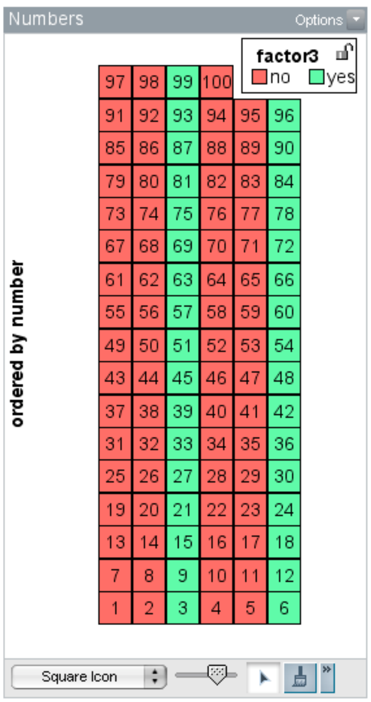
Have a few students present their answers to Steps 15 and 16.

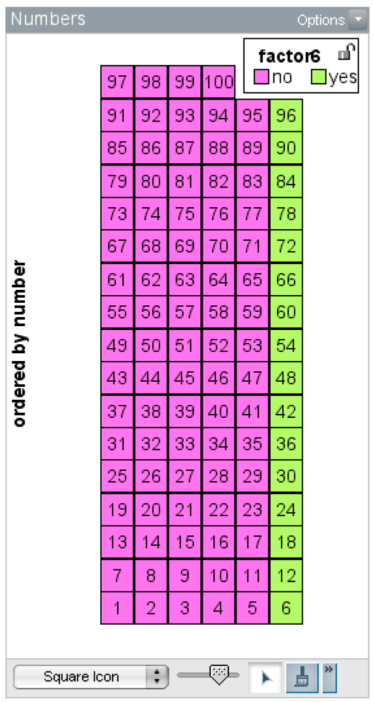
## Extensions (optional)



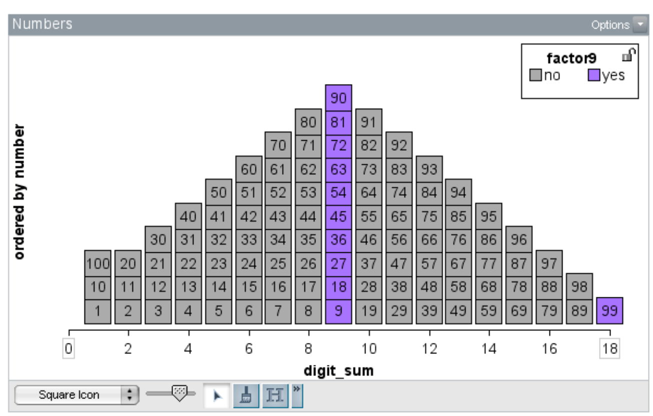

1. As written, this activity always uses a plot of stacked, ordered, square icons. Challenge students to experiment with other types of plots that show interesting patterns. For example, this ordered fused circle colored by *factor10* shows how multiples of 10 are evenly spaced throughout the numbers from 1 to 100.

2. Have students create two plots stacked and ordered by *number* and look for relationships between them. They can try two stacks of different widths colored by the same factor, or two stacks of the same width colored by different factors. Challenge students to explain why the relationships exist. For example, these two plots show that every multiple of 6 is also a multiple of 3, but only half of the multiples of 3 are multiples of 6. This is easily explained because 3 is itself a factor of 6. Students could even compare a third plot—*factor2*—and see that every multiple of 6 is also a multiple of 2, but only a third of the multiples of 2 are multiples of 6.



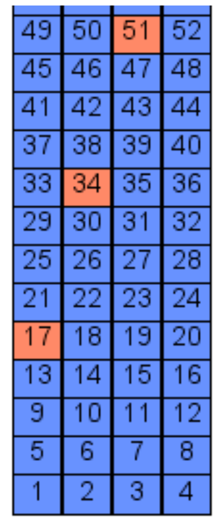




3. Students can begin to incorporate some of the other attributes such as *even*, *digit_sum*, or *ones_digit*. Exploring how these relate to the factor attributes can introduce or reinforce many division "rules," such as "all even numbers are divisible by 2" and "if a number ends in 0 or 5, it is divisible by 5." The plot below illustrates the rule "if the sum of the digits is a multiple of 9, then the number is divisible by 9."



4. Students in grades 6–8 who are beginning to learn about prime numbers can use the stacks and factor attributes to explore the concept of prime. Because a prime number only has factors of 1 and itself, the prime numbers will never appear in the last column of a stack, except for the first row. As an example, consider 17. If you make stacks 2, 3, 4, . . . , 15, 16, and 17 wide, the only time 17 will be in the last column is when the stack is 17 wide. So the only factors of 17 are 1 and 17, and 17 is prime.

5. Students in prealgebra or algebra could write algebraic expressions for the patterns of multiples in the stack. Consider this stack that is 4 wide and colored by *factor17*:




The first multiple of 17 is 1 more than the 4th row of 4, the second multiple of 17 is 2 more than the 8th row of 4, and so on. Students could write the sequence of numbers as

$$4 \times 4 + 1 = 17$$

$$4 \times 8 + 2 = 34$$

$$4 \times 12 + 3 = 51$$

$$4 \times 16 + 4 = 68$$

Advanced students could even generalize the sequence for the *n*th multiple of 17:

$$4 \times 4n + n = 17n$$

## ANSWERS

4. Answers will vary depending on grade level. If students have left the icons colored by *number*, they may notice only that the color gets darker as the numbers increase. Older students may notice patterns as they go up the columns, such as "3 is added each time I go up." They may also notice that the third column represents the multiples of 3.


6. Students should identify the third column as "multiples of 3." Some students may appropriately use the terminology defined at the beginning of the worksheet and say that these are the numbers with "3 as a factor."

7. Predictions will vary. An ideal answer is to guess that a stack 5 wide and colored by *factor5* will have all the multiples of 5, or the numbers with 5 as a factor, in the last column.

8. In general, a stack *X* wide and colored by *factorX* will have all multiples of *X*, or all numbers with *X* as a factor, in the last column. (*Note:* Most students will not be able to state this in these general terms, but they may be able to give several numeric examples.)

   After trying several width-factor combinations, students should be able to explain why this happens: "The numbers with 5 as a factor are all multiples of 5. So, if the stack is 5 wide, all the multiples of 5 will be on the end of the rows."

10. The icon for 24 will be colored like all of the numbers with 8 as a factor. This means that 8 is a factor of 24, and 8 divides into 24 with no remainder.

11. The icon for 24 is in the 3rd row of the stack. The 3rd row is related to the answer 3.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

13. This plot shows that there are 3 complete groups of 8 in 24 and this another way of explaining why 24 ÷ 8 = 3.

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

This solution represents division by what may be another familiar process—making equal groups. However, because the row of icons is continuous, some students may have trouble recognizing it as groups.

14. Students can use either method for these division problems. The answers below show alternating methods. The divisions with remainders will be more challenging, although not impossible. Whether students answer with fractions or remainders will depend on grade level and proficiency with division and fractions.

    a. 6; a stack 3 wide and colored by *factor3* shows 18 is in the 6th row.

    b. 2; a stack 26 wide and colored by *factor13* shows 13 goes in 2 times.

    c. $5\frac{1}{2}$, or 5 remainder 1; a stack 2 wide and colored by *factor2* shows 11 is 1 more than the 5th row, or $5\frac{1}{2}$ rows.

    d. $2\frac{2}{3}$, or 2 remainder 2; a stack 8 wide and colored by *factor3* shows that 3 goes in 2 whole times with 2 out of 3 left over.

16. Answers will vary.